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Dynamic behaviour of electron interference in the two-mode SU(1,1) coherent state field

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Abstract. Aharonov-Bohm-type electron interference in the presence of the two-mode SU(1,1) coherent state(CS) field is investigated. The visibility of the time-averaged interference pattern is discussed for this state, and a comparison with classical cases is made. It is shown that the time evolution of the intensity of electron interference exhibits collapse and revival (CR) phenomenon for this state. The fluctuation in electron interference, and its relation to CR phenomenon are also discussed.

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1 Introduction

The Aharonov-Bohm (AB) effect, as an unusual but important quantum effect, has been studied extensively, since its theoretical prediction in 1959 [1,2]. The effect is the production of a relative phase shift between two electron beams enclosing a magnetostatic flux, even if the electron beams do not touch the magnetic flux. The effects of enclosed fluxes often appear as observable changes in quantum interference patterns, although the fluxes may also affect the energy spectrum and kinetic momentum eigenvalues of the electrons. The AB effect is usually explained by means of the vector potential, which is present in multiply connected region of space where no magnetic induction field acts on the electrons. Whereas the charge and current densities are unique, the vector potentials are susceptible to gauge transformation. Nevertheless, the observable AB phase shifts are gauge invariant, depending only on the magnetic flux in the region from which the electron is excluded. Such an effect is inconceivable in classical physics and directly demonstrates the gauge principle of electromagnetism [3]. The AB effect has been detected [4,5] in interference experiments with electrons.

The convensional way of discussing electron interference is to treat the field classically, while the electron is treated quantum mechanically. Recently electron interference in the presence of nonclassical microwave fields with frequency ω_1, and classical radiation field with frequency ω_2, have been studied [6,7], and the visibility of the time-averaged intensity has been discussed similar to the Shapiro steps in the context of Josephson junctions [8], and also a comparison with the corresponding classical case has been made. In the case of a nonclassical electromagnetic field, the relative phase shift between the two electron beams is a quantum-mechanical operator, whose expectation value with regard to the density matrix describing the nonclassical electromagnetic field is time-dependent and has quantum fluctuations.

In the last few years there have been a lot of theoretical and experimental works on nonclassical electromagnetic fields [9], and there has been much interest in the production and properties of correlated two-mode states of the nonclassical electromagnetic field [10–12]. Such correlated two-mode states often display nonclassical characteristics such as squeezing and antibunching as well as violations of the Cauchy-Schwarz inequality and Bell inequality. The importance of correlated two-mode states of a field lies in their close connection to the two-photon nonlinear optical processes. In this article we are very interested in the effect of correlated two-mode states of a nonclassical electromagnetic field, specifically the correlated two-mode SU(1,1) coherent state(CS) [12], on the electron interference, and expect to find some interesting effects that have no counterpart in the case of the classical electromagnetic field. SU(1,1) CS can be generated from the action of a nondegenerate parametric amplifier on a two-mode state that contains q photons in one mode and none in the other [12]. Through numerical and analytical studies we find that the time evolution of electron intensity exhibits the collapse and revival (CR) phenomenon. The so-called CR phenomenon is well known in the context of the Jaynes-Cummings (JC) model in quantum optics [13–15]. A two level atom interacting with a nonclassical electromagnetic field is known as a JC model [13], in which Eberly *et al.* [14] have theoretically found CR in time evolution of atomic inversion, and evidence for CR has been found by Rempe *et al.* [15]

2 Shapiro steps in electron interference with SU(1,1) CS field

An experiment in the case of a nonclassical electromagnetic field at low temperature similar to the usual AB experiment was proposed in [6]. The requirement of low temperature makes the thermal fluctuations smaller than the quantum fluctuations. In this experiment a beam of electrons is split into two beams (for example by using an electrostatic biprism), each of them enters a waveguide through one hole and exits through another hole, in which the nonclassical electromagnetic field is travelling (see Fig. 2 in [6]). The magnetic field is perpendicular to the plane containing the paths of the two beams, while the electric field is in the plane.

2.1 Interference with classical and nonclassical magnetic flux

In the case of a two-mode nonclassical electromagnetic field, the electron can feel both an ac vector potential A and ac electric field E, according to the principle of superposition of the fields, which can be decomposed into two independent parts:

$$A = A_1 + A_2,\ E = E_1 + E_2 \quad (1)$$

where A_k and $E_k (k = 1, 2)$ is induced by ac nonclassical electromagnetic field mode with frequency ω_k. Integrating A_k and $E_k (k = 1, 2)$ in a closed loop gives the magnetic flux ϕ_k and external voltage V_k correspondingly, which are conjugated quantum variables satisfying

$$[\phi_k, V_j] = i\omega_k \delta_{k,j} \quad (j, k = 1, 2). \quad (2)$$

Thus the total magnetic flux ϕ and external voltage V induced by two-mode nonclassical electromagnetic field takes the form

$$\phi = \phi_1 + \phi_2, \quad V = V_1 + V_2. \quad (3)$$

Let Ψ_0, Ψ_1 be the wave functions at some point R corresponding to winding numbers 0, 1 respectively, in the absence of a nonclassical electromagnetic field. More complicated paths with other winding numbers are assumed to be negligible. In the case of a nonclassical electromagnetic field, the intensity of electrons at some point R in the observing region is [6]

$$I(R,t) = |\Psi_0|^2 + |\Psi_1|^2 + 2|\Psi_0\Psi_1| \times \mathrm{Re}\{\exp(i\sigma)\mathrm{Tr}[\rho\exp(ie\phi)]\} \quad (4)$$

where ρ is the density matrix describing the nonclassical electromagnetic field and $\sigma = \arg(\Psi_1) - \arg(\Psi_0)$. In this article we use the system of units in which $\hbar = c = k_\mathrm{B} = 1$, and the charge of the electron is dimensionless being equal to $e = \sqrt{4\pi/137}$.

The Hamiltonian of the nonclassical electromagnetic field in the two-mode case is

$$H = \sum_{j=1}^{2} \omega_j (a_j^+ a_j + \frac{1}{2}) \quad (5)$$

where a_j^+, $a_j (j = 1, 2)$ are the corresponding creation and annihilation operators of the nonclassical electromagnetic field with frequency $\omega_j (j = 1, 2)$, and satisfy the commutation relation $[a_k, a_j^+] = \delta_{k,j}$. Using Hamiltonian(5), we can get ϕ in the Heisenberg picture as follows

$$\phi(t) = 2^{-1/2}\sum_{j=1}^{2}[\exp(i\omega_j t)a_j^+ + \exp(-i\omega_j t)a_j]. \quad (6)$$

Here we assume that the electron currents are weak enough so that the external field is treated as free. Apart from the nonclassical electromagnetic field, a classical flux $V_0 t$ is also imposed which produces a static electromotive force V_0. This can be achieved by using a solenoid with a current that increases linearly as a function of time. Then the total magnetic flux becomes

$$\phi(t) = V_0 t + 2^{-1/2}\sum_{j=1}^{2}[\exp(i\omega_j t)a_j^+ + \exp(-i\omega_j t)a_j]. \quad (7)$$

For simplicity we assume that $|\Psi_0| = |\Psi_1| = 1/\sqrt{2}$. Then from (4) and (7) we get

$$I(R,t) = 1 + \mathrm{Re}\{\exp[i(\sigma + eV_0 t)] \times \mathrm{Tr}[\rho D_1[\frac{e}{\sqrt{2}}\exp(i\omega_1 t + \frac{i\pi}{2})]D_2[\frac{e}{\sqrt{2}}\exp(i\omega_2 t + \frac{i\pi}{2})]]\} \quad (8)$$

where $D(A) = \exp(Aa^+ - A^* a)$ is the displacement operator.

In reference [6] the magnetic flux which is the superposition of two sinusoidals with frequencies ω_1 and ω_2 has been considered. Two cases are discussed, one is that both modes are classical, and the other is that the mode with frequency ω_1 is nonclassical, and mode with frequency ω_2 is classical. It has been shown that the visibility of the time-averaged intensity is a constant for all irrational values of ω_1/ω_2, and show peaks (fractional Shapiro steps) at all rational values. In their discussion there is no correlation between the two modes. In this article we are interested in the effect of the correlation between the two modes on the dynamic behaviour of electron interference. Now we consider the two-mode SU(1,1) CS [12]. In the two-mode case the SU(1,1) Lie algebra may be realized as

$$K_0 = \frac{1}{2}(a_1^+ a_1 + a_2^+ a_2 + 1) \quad (9a)$$

$$K_+ = a_1^+ a_2^+ \quad (9b)$$

$$K_- = a_1 a_2. \quad (9c)$$

The SU(1,1) CS is defined as [12]

$$|\xi, q> = S_{12}(\beta)|q,0> = (1-|\xi|^2)^{(1+q)/2}\sum_{n=0}^{\infty}[\frac{(n+q)!}{n!q!}]^{1/2}\xi^n |n+q, n> \quad (10)$$

where

$$S_{12}(\beta) = \exp(\beta a_1^+ a_2^+ - \beta^* a_1 a_2) \tag{11}$$

is the two-mode squeeze operator, and $\beta = -\frac{1}{2} r \exp(-i\varphi)$, and $\xi = -\tanh(r/2)\exp(-i\varphi)$ with $0 < r < \infty$ and $0 < \varphi \leq 2\pi$. From (10) we can see that SU(1,1) CS are a superposition of the two-mode Fock states $\mid n_1 n_2 > \equiv \mid n_1 > \otimes \mid n_2 >$, in which the difference in photon numbers is always fixed, *e.g.*, $n_1 - n_2 = q$. The mean photon number N_1(mode 1) and N_2(mode 2) are

$$N_1 = q + N_2 \tag{12a}$$

$$N_2 = \frac{q+1}{2}(\cosh(2 \mid \beta \mid) - 1) = (q+1)\sinh^2(\mid \beta \mid) \tag{12b}$$

It is worth mentioning that the two-mode squeezed vacuum state is just a special case of the SU(1,1) CS for $q = 0$. The two-mode squeezed vacumm state has been widely studied in connection with nonclassical state of electromagnetic field [10–12]. The two-mode squeezed operator transforms the annihilation operators as follows

$$S_{12}^+(\beta) a_1 S_{12}(\beta) = \cosh(\mid \beta \mid) a_1 + \frac{\beta}{\mid \beta \mid}\sinh(\mid \beta \mid) a_2^+ \tag{13a}$$

$$S_{12}^+(\beta) a_2 S_{12}(\beta) = \cosh(\mid \beta \mid) a_2 + \frac{\beta}{\mid \beta \mid}\sinh(\mid \beta \mid) a_1^+. \tag{13b}$$

For a pure SU(1,1) CS it can be derived from (8,10,13) that

$$I(R,t) = 1 + \mathrm{Re}\{\exp[i(\sigma + eV_0 t)]\exp[-Y(t)] L_q[Y(t)]\} \tag{14}$$

where $L_q(x)$ are the Laguerre polynomials, and

$$Y(t) = \frac{e^2}{2}\{\cosh(2 \mid \beta \mid) - \sinh(2 \mid \beta \mid)\cos[(\omega_1 + \omega_2)t + \varphi]\}. \tag{15}$$

In deriving (14) we have used the formula

$$< q \mid D(A) \mid q > = \exp(- \mid A \mid^2 /2) L_q(\mid A \mid^2) \tag{16}$$

Using formula $\exp(A\cos\theta) = \sum_{n=-\infty}^{\infty} I_n(A)\exp(in\theta)$ we can expand (14) into

$$I(R,t) = 1 + \exp[-e^2\cosh(2 \mid \beta \mid)/2] \times \mathrm{Re}\{\sum_{n=-\infty}^{\infty} I_n[e^2 \sinh(2 \mid \beta \mid)/2] \times \exp\{i[(eV_0 + n(\omega_1+\omega_2))t + \sigma + n\varphi]\} L_q[Y(t)]\} \tag{17}$$

where $I_n(x)$ is the modified Bessel function. Using the following relations: $L_q(x) = \sum_{k=0}^{q} \binom{q}{k} (-x)^k/k!$ [16], $(a+b)^k = \sum_{l=0}^{k} \binom{k}{l} a^l b^{k-l}$, and $\cos x = (\mathrm{e}^{ix} + \mathrm{e}^{-ix})/2$, we can expand the Laguerre polynomials in (14), and then we can rewrite (14) as

$$I(R,t) = 1 + \mathrm{Re}\{\exp[-e^2\cosh(2 \mid \beta \mid)/2] \times \sum_{n=-\infty}^{\infty}\sum_{k=0}^{q}\sum_{l=0}^{k}\sum_{p=0}^{l} I_n[e^2\sinh(2 \mid \beta \mid)/2] \times \binom{q}{k}\binom{k}{l}\binom{l}{p}(\frac{1}{2})^l A^{k-l}B^l/k! \times \mathrm{e}^{i\{[(2p-l+n)(\omega_1+\omega_2)+eV_0]t+\sigma+(2p-l+n)\varphi\}}\} \tag{18}$$

where $A = -e^2\cosh(2 \mid \beta \mid)/2$, $B = e^2\sinh(2 \mid \beta \mid)/2$. It is easy to see from (18) that when the following condition is satified

$$eV_0 = N(\omega_1 + \omega_2) \tag{19}$$

where N is an integer, we can take the time average of (18) and get

$$\overline{I(R)} = 1 + \exp[-e^2\cosh(2 \mid \beta \mid)/2] \times \sum_{k=0}^{q}\sum_{l=0}^{k}\sum_{p=0}^{l} I_{l-N-2p}[e^2\sinh(2 \mid \beta \mid)/2] \times \binom{q}{k}\binom{k}{l}\binom{l}{p}(\frac{1}{2})^l A^{k-l}B^l/k!\cos(N\varphi - \sigma) \tag{20}$$

Because the phase difference σ is a function of R, it is easy to see from (20) that as R moves along the screen we get the interference fringes, and the visibility of the corresponding fringes is

$$\alpha = \exp[-e^2\cosh(2 \mid \beta \mid)/2] \times \sum_{k=0}^{q}\sum_{l=0}^{k}\sum_{p=0}^{l} I_{l-N-2p}[e^2\sinh(2 \mid \beta \mid)/2] \times \binom{q}{k}\binom{k}{l}\binom{l}{p}(\frac{1}{2})^l A^{k-l}B^l/k! \tag{21}$$

A plot of α *versus* $eV_0/(\omega_1+\omega_2)$ will have peaks when (19) is satisfied. When (19) is not satisfied, time averaging of electron intensity will be a constant, being equal to 1, so $\alpha = 0$, *i.e.*, the complete destruction of the interference pattern. (19) can be understood as the energy that the electron looses due to the classical electromagnetic field, compensated by N photons mode with frequency ω_1, and N photons mode with frequency ω_2 simultaneously.

2.2 Interference with nonclassical magnetic flux only

When the classical magnetic flux is not imposed, *i.e.*, $V_0 = 0$, one can easily get the time-averaged electron interference intensity from (18) with $V_0 = 0$.

$$\overline{I(R)} = 1 + \exp[-e^2\cosh(2 \mid \beta \mid)/2] \times \sum_{k=0}^{q}\sum_{l=0}^{k}\sum_{p=0}^{l} I_{l-2p}[e^2\sinh(2 \mid \beta \mid)/2] \times \binom{q}{k}\binom{k}{l}\binom{l}{p}(\frac{1}{2})^l A^{k-l}B^l/k!\cos(\sigma) \tag{22}$$

and the corresponding visibility is

$$\alpha = \exp[-e^2 \cosh(2 \mid \beta \mid)/2] \times \sum_{k=0}^{q}\sum_{l=0}^{k}\sum_{p=0}^{l} I_{l-2p}[e^2 \sinh(2 \mid \beta \mid)/2] \times \binom{q}{k}\binom{k}{l}\binom{l}{p}(\frac{1}{2})^l A^{k-l}B^l/k!. \quad (23)$$

From (22-23) one can see that the results for SU(1,1) CS are quite different from the results for the two-mode classical case, and the cases of one mode classical and one mode nonclassical [6]. In the later two cases a plot of α *versus* ω_1/ω_2 will be constant for all irrational values, and it will have peaks at the various rational values. The physical explanation is frequency conversion from N photons with frequency ω_1 to M photons with frequency ω_2 [6]. One can easily see from (23) that for SU(1,1) CS α is a constant independent of both ω_1 and ω_2. The frequency conversion between frequencies ω_1 and ω_2 is forbidden here. For SU(1,1) CS the two modes are strongly correlated, *e.g.*, the difference between the number of photons in mode ω_1 and mode ω_2 is fixed to q. In this case only an equal number of photons of mode with ω_1 and of mode with ω_2 can be absorbed or emitted by electrons simultaneously.

In a convensional AB experiment a relative phase shift between two electron beams enclosing a magnetostatic flux generally produces observable fringe shifts in the interference pattern. One can see from (22-23) that in the case of SU(1,1) CS field without the classical flux, *i.e.*, $V_0 = 0$, there is no phase shift, and so there is no observable fringe shift in the time-averaged interference pattern, only the visibility is changed.

3 Dynamics of electron interference

The fully quantum mechanical model problem involving the interaction of a radiation field with matter has long been a concern. The JC model is just a typical one. In this simple model CR in the dynamic behaviour of atomic inversion has been found both theoretically and experimentally [13–15]. We have found that such a pure quantum effect also exists in supercurrent in a mesoscopic Josephson junction with nonclassical electromagnetic field [17]. In this section, we will show that the dynamic behaviour of electron interference with SU(1,1) CS field also exhibits CR phenomenon.

We plot the time evolution of electron intensity versus the scaled time $(\omega_1 + \omega_2)t/\pi$ that shows the typical periodic CR phenomenon in Figure 1. Mathematically CR can be easily understood from (14-15). One can see from (15) that $Y(t)$ is non-negative and is an oscillating function with period of $2\pi/(\omega_1 + \omega_2)$. Thus the second term in (14), which determines the interference pattern, is weighted with a time-dependent factor which periodically suppresses the coherence of the electrons. This factor is closely related to the quantum fluctuation of the nonclassical electromagnetic field and causes partial destruction of

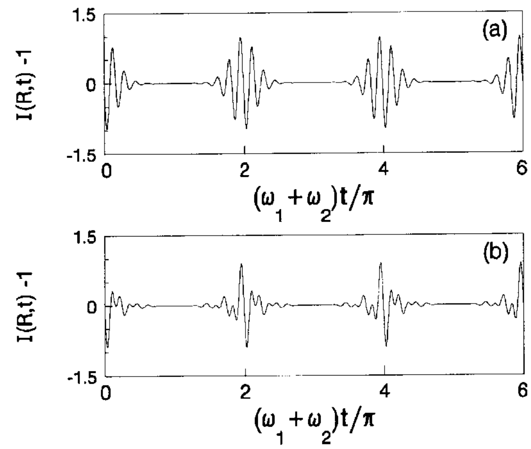

Fig. 1. Electron interference intensity *versus* scaled time $(\omega_1 + \omega_2)t/\pi$ for $\sigma = \pi/2$, $\varphi = 0$, $eV_0/(\omega_1 + \omega_2) = 12$, $\mid \beta \mid= 2.5$, and for (a) $q = 0$; (b) $q = 3$.

the electron interference. The maximum of $Y(t)$ is $Y_{\max} = e^2[\cosh(2 \mid \beta \mid) + \sinh(2 \mid \beta \mid)]/2$ for fixed $\mid \beta \mid$. For very small $\mid \beta \mid$ the minimum of $\exp[-Y(t)](= \exp(-Y_{\max}))$ is almost equal to 1, the suppression is very weak, and the oscillations of electron intensity appear to be almost regular, with no true CR. As $\mid \beta \mid$ is increased, we get the imcomplete collapse. When $\mid \beta \mid$ is large enough, CR occurs with complete collapse, as Figure 1 shows, and now $\cosh(2 \mid \beta \mid) \simeq \sinh(2 \mid \beta \mid) \simeq \exp(2 \mid \beta \mid)/2$, the collapse function can be found from (14-15) as

$$\exp\{-\frac{e^2}{2}\exp(2 \mid \beta \mid)\sin^2[\frac{(\omega_1 + \omega_2)t + \varphi}{2}]\}.$$

We have confirmed the above results with numerical calculation, and noticed that with the increased $\mid \beta \mid$ CR becomes more and more compact and distinct, and the time between revivals also increases.

For $m = 0$, *i.e.* the two-mode squeezed vacuum state, the frequency is eV_0 during the revivals, which means that perfect and complete oscillations are indeed essentially sinusoidal as Figure 1a shows. As m is increased, the spread in relevant photon numbers also increases [12], and the expected increasingly irregular behaviour of the oscillations is observed, see Figure 1b.

Physically CR phenomenon of electron interference can be explained as a consequence of quantum interference in phase space. One can see that the summation in (18) represents oscillations with different freqencies due to the effect of the electromagnetic fields, both classical and nonclassical. Because different components in the summation oscillate with different frequencies, they will become decorrelated, *i.e.*, the collapses are due to the destructive interference of oscillations with different frequencies in (18). The revivals are a manifestation of the quantum nature of the nonclassical electromagnetic field, which is mathematically reflected in the discrete summation; that is the evolution of the electron intensity is determined by the individual field quanta. The discrete characteristic ensures that after some finite time all oscillating terms almost come back in phase with each other, restore the

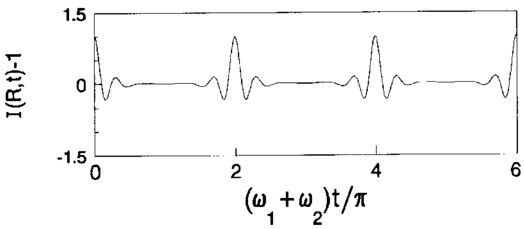

Fig. 2. Electron interference intensity *versus* scaled time $(\omega_1+\omega_2)t/\pi$ for $\sigma=0$, $\varphi=0$, $V_0=0$, $\mid\beta\mid=2.5$, and $q=8$.

coherent oscillations, and give an appreciable value to electron intensity. The rephasing is perfect when the ratio $eV_0/(\omega_1+\omega_2)$ is rational, *i.e.*, the frequencies in (18) are commensurate; and is not perfect when the ratio is an irrational.

When there is no classical magnetic flux present, *i.e.*, $V_0=0$, the electron intensity can also exhibit CR phenomenon for this state. When the nonclassical electromagnetic field is not present, the classical magnetic flux V_0t will produce an oscillation in electron interference with frequency eV_0. When both V_0t and nonclassical magnetic flux are present, the interaction between the oscillation with frequency eV_0 and the oscillation due to the nonclassical magnetic flux leads to the appearance of CR phenomenon and the time-averaged interference pattern with proper value of V_0. But when $V_0=0$, the nonclassical magnetic flux brings about an oscillation with the basic frequency of the external field and its harmonics, which can be easily seen from (18) with $V_0=0$. In this case it is the contribution of the coherence of the infinite harmonics which bring about CR phenomenon, as Figure 2 shows, and CR is entirely due to the coherence of the external field.

Noting that σ is a function of R, it can be inferred from (20) that for $\varphi=0$ when $\sigma(R_1)=0$, $\overline{I(R_1)}$ reaches its maximum $1+\alpha$; when $\sigma(R_0)=\pi/2$, $\overline{I(R_0)}=1$; and when $\sigma(R_2)=\pi$, $\overline{I(R_2)}$ reaches its minimum $1-\alpha$, where α is the visibility. In Figure 3 we plot the time evolution of $I(R,t)$ for $\sigma(R_1)=0$, $\sigma(R_0)=\pi/2$, and $\sigma(R_2)=\pi$. From Figure 3 one can see that $I(R,t)$ simultaneously collapse to 1 in the collapses period independent of $\sigma(R)$, that means although the time-averaged interference pattern is unchanged, the interference fringe completely disappears in this period. This can be understood from (14-15) that when $\mid\beta\mid$ is large and $(\omega_1+\omega_2)t$ is around $(2n+1)\pi$, n integer, the suppressing factor $\exp[-Y(t)]$ becomes exponentially small, and then $I(R,t)$ collapses to 1, independent of σ. While in the revival period the time-averaged electron intensities over this period are bigger than 1 for $\sigma(R_1)=0$ (see Fig. 3a), equal to 1 for $\sigma(R_0)=\pi/2$ (see Fig. 3b), and less than 1 for $\sigma(R_2)=\pi$ (see Fig. 3c), that means the interference pattern reappears. And then this process is periodically repeated. So if we are able to observe the time-dependent interference pattern, which might be realized with a clever stroboscopic measurement in the spirit of [18], we would be able to directly observe the CR in the experiment.

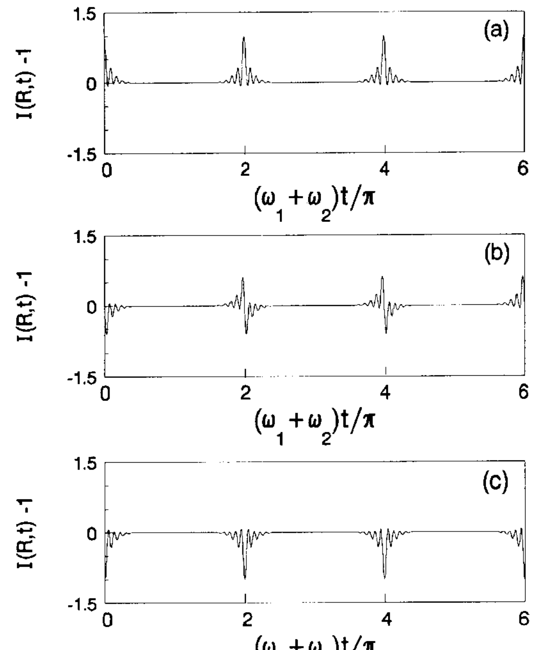

Fig. 3. Electron interference intensity *versus* scaled time $(\omega_1+\omega_2)t/\pi$ for $\mid\beta\mid=3$, $\varphi=0$, $eV_0/(\omega_1+\omega_2)=12$, $q=8$, and for, (a) $\sigma=0$; (b) $\sigma=\pi/2$; (c) $\sigma=\pi$.

4 Quantum fluctuations

Electron interference with a classical magnetostatic flux has been fully discussed, and in this case the flux is definite without fluctuations. In the case of a nonclassical electromagnetic field, the flux is a quantum operator that has quantum uncertainty in its values. It is the quantum fluctuations of flux ϕ that will partially destroy the electron interference. To see how the quantum noise of a nonclassical electromagnetic field affect the phase shift and then the corresponding electron interference, in this section we are interested in the quantum fluctuations of the operator $f\equiv\mathrm{Re}\{\exp[i(e\phi+\sigma)]\}$. It is easy to see from (4) that the expectation value of f determines the dynamics of the electron interference. We will show in the following that the CR phenomenon of electron interference is closely related to the fluctuation of such an operator. The fluctuation of the operator f is defined as

$$\begin{aligned}<(\Delta f)^2> &\equiv < f^2 > - < f >^2\\ &= \frac{1}{2}+\frac{1}{2}\mathrm{Re}\{\mathrm{Tr}[\rho\exp(2ie\phi+2i\sigma)]\}\\ &\quad -\{\mathrm{Re}\{\mathrm{Tr}[\rho\exp(ie\phi+i\sigma)]\}\}^2. \end{aligned} \tag{24}$$

For SU(1,1) CS from (7,13,16,24) we can get

$$\begin{aligned}\langle(\Delta f)^2\rangle =&\frac{1}{2}+\frac{1}{2}\exp[-4Y(t)]L_q[4Y(t)]\cos(2eV_0t+2\sigma)\\ &-\exp[-2Y(t)]L_q^2[Y(t)]\cos^2(eV_0t+\sigma). \end{aligned} \tag{25}$$

At this point it is very interesting to compare (14) and (25). When $\mid\beta\mid$ is large and $(\omega_1+\omega_2)t$ is around $(2n+1)\pi$,

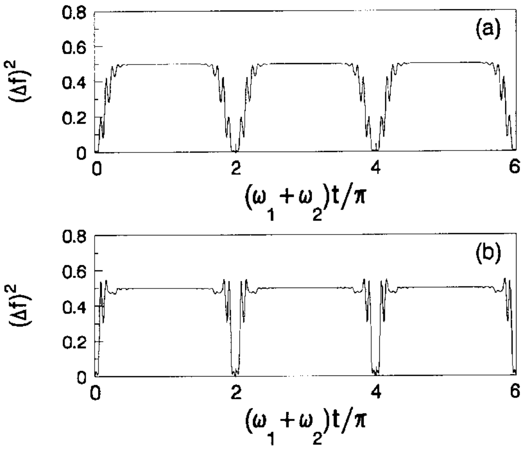

Fig. 4. Quantum fluctuations of the operator f, $< (\Delta f)^2 >$, *versus* scaled time $(\omega_1 + \omega_2)t/\pi$ for the same parameters as Figure 1.

n integer, the suppressing factor $\exp[-Y(t)]$ in the second term in (14), and $\exp[-4Y(t)]$, $\exp[-2Y(t)]$ in the second and third term in (25) respectively become exponentially small, and $I(R,t)$ collapses to 1 and $< (\Delta f)^2 >$ reaches its steady value 0.5. In Figure 4 we display the time evolutions of quantum fluctuations of operator f for the same parameters as Figure 1 for SU(1,1) CS. The time evolution of the electron interference brings about fluctuation reduction in f, the minimum fluctuations of f is occuring when the electron intensity revives to its maximum; the maximum of fluctuations is achieved at the period between two revivals, where electron intensity reaches its steady value equal to 1 at any R on the screen (now $I(R,t)$ is independent of σ or R), that is the complete destruction of the interference fringe. This can be easily understood, because the fluctuations have their origin in destructive quantum interference of different oscillations, *i.e.*, a rise in fluctuations means that different oscillations begin to partially lose their correlations, and when the fluctuations are smallest, different oscillations are mostly correlated.

5 Conclusion

We have investigated the dynamic behaviour of Aharonov-Bohm-type electron interference in the presence of the two-mode SU(1,1) CS field, with or without classical magnetic flux. In this case the relative phase shift between the two electron beams is a quantum operator and its expectation value determines the dynamics of electron interference. When $V_0 \neq 0$ we have found that the time-averaged interference fringes exist only for special values of V_0, *i.e.*, $V_0 = n(\omega_1 + \omega_2)/e$, n integer, which has been called as the voltage steps similar to the Shapiro steps in the context of Josephson junctions. When $V_0 = 0$, *i.e.*, without classical magnetic flux, we have found that there is no frequency conversion between the two modes for SU(1,1) CS, which is quite different from the results for uncorrelated two classical modes, and the results for one classical mode and one nonclassical mode [12]. This results from the fact that the two modes in SU(1,1) CS are strongly correlated.

We have also shown that the dynamic behaviour of the electron interference exhibits CR for SU(1,1) CS. We emphasize here that CR is a purely quantum effect which is due to the qunatum nature of the field. We also have shown that CR of electron interference is closely related to the fluctuation of a nonclassical electromagnetic field. By studying the fluctuation of the operator $f = \mathrm{Re}\{\exp[i(e\phi + \sigma)]\}$, we have found that the minimum fluctuations of the operator f occur for SU(1,1) CS when the electron intensity revives to its maximum, and the maximum fluctuation is achieved when the electron intensity collapses to 1. The complete collapse leads to the complete destruction of the interference fringe. This is so because the fluctuations have their origin in the destructiveness of quantum interference. We hope the results obtained in this paper could be confirmed experimentally in the future.

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